An Optimal Islamic Investment Decision in Two-region Economy: The Case of Indonesia and Malaysia

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Abstract

In this work, the possibility of cross-border activities between two regions in the framework of the investment contract is viewed as optimal allocation problems. The problems of determining the optimal proportion of funds to be invested in liquidity and technology are analyzed in two different environments. In the first case, we consider a two-region and two-technology economy in which both regions possess the same productive technology or project, but a different stream of return. While in the second case, we examine an economy where two regions (Indonesia and Malaysia) hold different Islamic productive projects with identical returns. Allocation models are formulated in terms of investors' expected utility maximization problem under budget constraints with respect to regional and sectoral shocks. It is revealed that optimal parameters for liquidity ratio, technological investment profile, and bank repayment are analytically characterized by the return of a more productive project and the proportion of impatient and patient investors in the region. Even though both cases employ different assumptions, they provide the same expressions of optimal parameters. The model suggests that cross-border Islamic investment activities between two regions might be realized, provided both regions hold productive projects with an identical stream of return. This paper also shows that by increasing the lower return of the project approaching the higher return, a room for inter-region investment can be created. An analytical framework of an investment contract in terms of optimal allocation model is provided.

Keywords: Investment contract; Optimal allocation model; Two-region economy.
JEL Classifications: C61, F36, G11.

1. Introduction

How should we invest our funds? Making an investment decision can be a tough task as it depends on many aspects, such as goal, time frame, and risk appetite. In financial management, investment decisions and capital budgeting are interchangeably used, and they refer to the allocation of money or other resources at a different time in expectation of economic returns in future periods. Due to the nature of uncertainty, an investment decision should be undertaken such that it considers lower risk and higher return appetites in the investment portfolio. In obtaining the right decisions, many financial planners interweave several techniques in clinical and actuarial decision making, i.e., from subjective judgments to mechanical and systematic algorithms (Jones, 2014). Any investment decision should be made based on subjective and objective factors (Virlics, 2013).

Growing works of research on investment decision behavior vary across dimensions. The first strand of the literature emphasizes the mechanical algorithm. Markowitz (1952) provided a guideline for selecting the most efficient portfolio based on the mean-variance model. Hirshleifer (1958) addressed two-period and multi-period investment problems and solved for the optimal scale and investment portfolio through the use of isoquant analysis. Diamond & Dybvig (1983) proposed an investment contract that can prevent bank runs and provide optimal risk-sharing by converting illiquid assets. Luban (2002) utilized an integrated approach that involves the use of Monte Carlo simulation to generate the probability distribution of NPV, stochastic dominance criterion to map the risk profiles, and utility theory to perform sensitivity analysis under different levels of risk aversion. Wong & Eng (2017) used the framework of the new Keynesian DSGE model and a macroeconomic model of shared-responsibility to inspect the stability of the Islamic financial contract. It was found that payoff distribution between bank and entrepreneur is dependent on the macroeconomic circumference via the entrepreneur's leverage, while that between investors and the bank is endogenous to the bank's capital and leverage. An option game theory course was applied by Wang & Chen (2011) to offset the intrinsic limitations of the conventional NPV method in investment decision making of a circular economy. Bielecki et al. (2005) exploited the so-called Bellman equations to describe the optimal investment pattern in maximizing expected constant relative risk aversion utility function.

Another strand of the literature relates to a question of how personal subjective factors determine investment decisions. It has been verified by Raut & Kumar (2018) that two groups of individual investors, namely experienced and newbie, exhibited different perceptions about behavioral factors such as informational cascades, herding, anchoring, and overconfidence in investment decision making. Areiqat et al. (2019) also confirmed that overconfidence, loss aversion, and herding have an impact on the stock investment decision making. Gill et al. (2018) discovered the fact that economic expectations and overconfidence bias have a significant influence on the investment decision making process among investors. An interesting fact was revealed from extensive survey research conducted by Lan et al. (2018), where demographic characteristics, such as age, gender, years of education, occupation, investment experience, financial knowledge level, and income, are closely linked to decision behaviors. Alshamy (2019) employed structural equation modeling to show that expertise, risk aversion, corporate governance, financial information, and experience to be significant determinants affecting investment decision making, in addition to gender, age, and financial education as moderating variables. A study by Schwarzkopf (2003) revealed that the attraction effect, i.e., an effect that occurs in which adding an inferior or irrelevant alternative into an existing choice set changes decision maker's perception due to its ability to increase the attractiveness, can influence investment decisions. In investment-based crowdfunding, it was found that investors with more extensive social networks invest more as social interactions relate closely to asymmetric information reduction. From risk aversion perspective, women invest less equity as the riskiest investments but more in bonds as safer ones (Herve et al. 2019).

On the other hand, a persistent-increasing trend of international economic integration has not only led to a broader and richer discussion on how investors should invest their funds across existing investment instruments and period but also on how investors optimally allocate their money across regions/countries. For instance, Nicoletti *et al.* (1976) discussed the resource allocation of investment between two regions with different characteristics in the economy within the framework of optimal control problems. A macroeconomic model was developed to determine optimal proportions of investment in each region that maximize social welfare criterion, i.e., the capital stock and the social consumptions, subject to dynamical state equations of production. Beyond its simplicity, an approach involving only two economies has proved pivotal in spatial economic modeling as it can provide detailed analysis results. Fecht, Gruner, and Hartmann (2012) exploited an optimal investment allocation model between two regions with different specializations to illustrate the risks and benefits of financial integration.

They argued that by diversifying risk, banks, which are one of the industries in the financial sector, can further enhance the resilience of their financial systems to shocks. Allen and Gale (2000) and Freixas, Parigi, and Rochet (2000) also utilized an optimal investment allocation model to demonstrate that financial integration through the interbank market allows the diversification of regional liquidity shocks efficiently while entailing the risk of financial contagion between banks from different regions.

Furthermore, voluminous works of research also illustrated the benefits and risks of financial integration. Choi and Cook (2011) also emphasized the benefits of stabilizing the integration of international financial accelerators that help diversify the effects of shocks between countries. The well-known Solow model has been carefully utilized by Gumpert (2019) to investigate regional economic disparities between two regions with different characteristics of technological acquisition, namely industrial and agricultural regions. It was demonstrated that inter-region financial transfers reduce the income gap between the advanced and less developed regions. Imbs (2006) also revealed that a financially integrated economy has a relatively stronger business movement. Cooperation between countries in the financial sector not only has an impact on reducing the risk of the impact of the crisis but also increases the openness of a country to other countries in the economic aspect, which reduces the role of government.

In this paper, we strive to encompass the previous works of literature in several crucial ways. First, this paper extends the optimal investment allocation model between two regions to not only explain a condition in which financial integration is possible (Allen and Gale, 2000; Freixas et al., 2000 Fecht *et al.,* 2012) but also exploit the model to illustrate a condition in which financial integration may not be possible between these regions. While Fecht *et al.* (2012) assumed that the two regions have their specialization on a particular investment instrument, it may not always be the case under certain circumstances. For instance, in regions with Islamic banking, *Mudaraba* (i.e., profit-sharing and loss-bearing) is known as a more productive project than *Murabaha* (i.e., cost-plus). However, *mudaraba* burdens the Islamic bank with a higher transactional cost as it contains more risks (Abdullah and Chee, 2014). In this case, the cost may be higher in poor and emerging economies as the investors should engage closely and frequently to the project they invest in order to avoid loss of the projects. Therefore, the net return provided by *mudaraba* may generally lower than *Murabaha*. In the case of Mudaraba, we can find many similar phenomenon moreover in the emerging countries like Indonesia and Malaysia – the countries we use as the samples in this research. In Indonesia, sometimes the banks as investor's representatives should pay more costs to give trainings and frequent surveillance to their respected project's investors. Besides, we also provide some recommendation to translate the non-financially-integrated regions to financially-integrated; therefore, leads to a specific region's specialization.

By extending the mathematical approach developed by Fecht *et al.* (2012), this paper explicitly examines decision making process in an investment contract between the bank and an investor in an economy consisting of two regions where cross-border activities may be conducted under uncertainty of the timing of project cash-flow realization due to shocks. We consider two different scenarios of investment contract to analyze the bank investment decision making in short-term liquidity and long-term projects. In the first case, two regions have similar productive projects but a different stream of returns. For the second case, two regions have different islamic productive projects but an identical stream of returns.

Under the assumption that the bank invests only in productive projects, it has been revealed that at the optimal level, both cases provide the same amounts of liquidity ratio, contract repayment, and expected utility. In the first case, it suggests that funds devoted to financing long-term projects should all be invested in productive projects in the same region of banks. While in the second case, the bank has the flexibility to invest the funds not only in productive projects in the same region but also in another region as cross-border activities are allowed in this case. This paper also shows that by increasing the lower return of the project approaching the higher return, a room for inter-region investment can be created. Cross-border activities thus can be seen as incentives for integration between the two economies.

Recall that this paper shed light on the possibility of integration between two regions in the framework of the investment contract is viewed as optimal allocation problems. The model suggests that integration between two regions might be realized provided both regions hold productive projects with an identical stream of return. Therefore, it also implies that, for the banking sectors, this paper could provide a framework to calculate optimum allocation. For the policymakers, this paper gives an academic-basis analysis for the policymaking in banking sectors. For the academic environment, this paper induces the academicians to develop studies concerning the optimum allocation, financial integration, and development of banking model with other relevant scenarios.

The rest of this paper is organized as follows. After some introduction and literature review in the previous section, we provide in Section 2 the description of the investment model under consideration. In Section 3, we present our findings on the optimal investment decision and discuss some implications. We conclude in Section 4.

2. The Investment Model

To depict the uncertainty of optimal investment contract between a financial intermediary, i.e., the bank, and an investor, we adopt the so-called Diamond-Dybvig model on liquidity demand (Diamond & Dybvig, 1983) and revisit an allocation model with integrated bank developed by Fecht *et al*. (2012). Investment contracts in an open economy or sector which consists of households as investors and banks as financial intermediary is considered within three periods of time: t_0, t_1, t_2. The economy is divided into two regions, namely region A (Indonesia) and region C $(Malaysia)$, where in each region there are only two illiquid investment projects or technologies, namely project R $(Mudaraba)$ and project S $(Murabaha)$. Project R promises investors a return of $R_A > 1$ per unit invested if it is run in region A and $R_C > 1$ if it is run in region C, while project S provides returns $S_A > 1$ and $S_C > 1$ per unit invested, respectively.

2.1 Investors

Investors are *ex ante* identical and assumed to be risk neutral, i.e. they are indifferent to the choices of investment projects that provide the same return, but one project may be riskier (i.e. Mudaraba). Investors invest their entire t_0 endowment, 1 per investor, and they have no further resources at periods t_1 and t_2. Toward their preference in investment, investors are divided into two groups, namely impatient and patient investors. The former is an investor that will leave the project in the event of late payment due to shocks in the period t_1 and choose to invest in a private investment with a marginal utility of X. The later chooses to wait until the period t_2 with marginal utility of 1, assuming that $X > 1$. In other words, impatient investors invest at t_1 only, while patient investors invest at either t_1 or t_2. Impatient investors is assumed to dominate the economy with a proportion of $r > 1/2$. Thus, the proportion of patient investors is equal to $(1 - r) < 1/2$. We assume that investors have the same expected utility toward investment return, namely $EU(c_1, c_2) = c_1 + c_2$, with c_1 and c_2 representing the expected returns earned in periods 1 and 2, respectively.

2.2 Banks

There is only one bank in each region and they operate in their respective regions. Banks are allowed to make cross-border activities. In period t_0, investors deposit their fund and bank invests the funds into short-term liquid activity, i.e., liquidity, and/or long-term illiquid projects. Investment in liquidity yields 1 per unit invested and long-term project yields either S_A, S_C, R_A, or R_B at period t_2, and nothing at period t_1. Banks are competitive and offer contracts to investors to maximize their expected utility. The contract promises payments of δ_1 at period t_1 to impatient investors and a late payment of δ_2 at period t_2 to patient investors. However, if bank has not enough funds to repay δ_1 and δ_2, then all investors exit from contract and withdraw their money at t_1. In this case, the bank will be liquidated. Since there exists liquidity risk due to uncertainty over the time of the payment of contracts, it is possible for bank to invest funds in a storage technology.

2.3 Shocks

The existence of financial integration enables the diversification of probability and the potential for financial contagion due to shocks in regions. We assume that the economy is at risk of two types of shocks, namely sectoral and regional shocks. If sectoral shocks attack either project R or project S, then the respective project in both region A and region C will be affected. If regional shocks attack either region A or region C, then both projects R and S in the respective region will be affected. Payment of the return on the project affected by shocks will be delayed, i.e., it is paid at period t_2. The probability distribution of sectoral and regional shocks is detailed in Table 1.

Table 1. Probability distribution of shock (Fecht *et al.*, 2012)

State	$(R_A; S_A)$	$(S_A; R_A)$	$(0; R_A + S_A)$
$(R_C; S_C)$	p	0	q
$(S_C; R_C)$	0	p	q
$(0; R_C + S_C)$	q	q	0

In Table 1, the rows represent region C, while the columns represent region A. We denote by $(R_C; S_C)$ a state where the return of project R in region C, namely R_C, is paid at t_1, while the payment of return of project S in region C, namely S_C, is delayed (due to sectoral shocks) so that it is paid for in period t_2. Notations $(S_C; R_C)$, $(R_A; S_A)$, and $(S_A; S_A)$ should be similarly interpreted. We denote by $(0; R_A + S_A)$ a state where a regional shock attacks region A such that the payment of returns of projects R and S, which is equal to $R_A + S_A$ in total, experiences delay and is paid for in period t_2. There is no payment in period t_1. Notation $(0; R_C + S_C)$ is interpreted similarly.

Therefore, the $(1,1)$-th element of the table indicates that there is a probability p for project S to be hit by sectoral shocks so that the payments of project S contract in both regions are deferred to period t_2, while it is carried out at t_1 for unaffected project R. The $(2,2)$-th element illustrates the situation of project R hit by sectoral shocks with probability p. The $(1,3)$-th and $(2,3)$-th elements assume that there is a probability of q for region A to be hit by regional shocks so that the payments for projects R and S projects in region A are late and thus are carried at period t_2, while only one project is late in region C due to sectoral shocks. The $(3,1)$-th and $(3,2)$-th elements assume that there is a probability of q that region B is affected by regional shocks. Meanwhile, elements $(1,2)$-th and $(2,1)$-th show no possibility of sectoral shocks occur with partial effects, and element $(3,3)$-th explains that it is not possible for regional shocks to attack both regions simultaneously. Since Table 1 considers all the states that might occur, thus it is satisfied that

$$2p + 4q = 1. \tag{1}$$

3. Findings

3.1 Optimal allocation problem

We mean by optimal allocation problem, a problem dealt with by the bank in determining proportion of funds to be invested in storage technology, i.e., liquidity, and projects. Instead of considering an ideal situation as discussed in Fecht *et al.* (2012), we devise a two-region economy where the same project is more productive than another in both regions, but with different return. In particular we consider projects R (mudaraba) and S (murabaha) in regions A (Indonesia) and C (Malaysia) where

$$S_A > S_C, R_A > R_C, S_A > R_A, S_C > R_C. \tag{2}$$

By (2) we assume project S (murabaha) is more productive in term of net return than project R (mudaraba) in both regions A (Indonesia) and C $(Malaysia)$ However, this project promises different stream of return with respect to regions: project S provides a higher return if implemented in region A (Indonesia) than in region $C.(Malaysia)$. The last assumption applies also to project R. This environment is commonly found in most regions with some member countries which implement Islamic banking like Indonesia (A) and Malaysia (C), where *mudaraba* (profit-sharing and loss-bearing) is known as a less productive project than *murabaha* (cost-plus). It is because *mudaraba* has higher transaction cost than *murabaha, mudaraba's net return in most cases is lower than murabaha's one*. Yet the return provided by murabaha/mudaraba varies between countries.

Let assume that we are in region A. Suppose that local bank invests funds in liquidity with proportion k and in projects with proportion $1-k$ and it is assumed that bank invests only in more productive project, i.e., project S in region A with proportion α and project S in region C with proportion $1-\alpha$. The repayment to investors with respect to shocks is described in Table 2.

Table 2. Repayment with respect to shocks to project S

Element	Shock		Probability	Repayment	
	A	C		t_1	t_2
$(1,3)$	RS	SS	q	rXk	$\alpha S_A(1-k) + (1-\alpha)S_C(1-k)$
$(2,3)$	RS	×	q	rXk	$\alpha S_A(1-k) + (1-\alpha)S_C(1-k)$
$(1,1)$	SS	SS	p	$rX\delta_1$	$(1-r)\cdot 1\cdot \delta_2$
$(2,2)$	×	×	p	$rX\delta_1$	$(1-r)\cdot 1\cdot \delta_2$
$(3,1)$	SS	RS	q	$rX\delta_1$	$(1-r)\cdot 1\cdot \delta_2$

(3,2)	×	RS	q	$rX\delta_1$	$(1-r)\cdot 1\cdot \delta_2$

RS: regional shock, SS: sectoral shock, ×: no shock

Regional shock will affect all projects in a region and force impatient investors out of contract. If regional shock attacks region A, then the bank will be liquidated and will be only able to repay its impatient investors per capita liquidity holding k. If sectoral shock hits project S, the bank can still repay δ_1 to impatient investors in period t_1 and δ_2 to patient investors in period t_2. Thus, based on Table 2, the investors' expected return at periods t_1 and t_2 are given by

$$c_1 = 2qrXk + (2p+2q)rX\delta_1. \quad (3)$$
$$c_2 = 2q(\alpha S_A + (1-\alpha)S_C)(1-k) + (2p+2q)(1-r)\delta_2. \quad (4)$$

Recall that impatient investors may earn a marginal utility of X by accepting a private investment after leaving the contract and patient investors may reserve a marginal utility of 1 by waiting up to period t_2.

Total investors' expected utility EU should be maximized is then constructed by augmenting expected returns in (3) and (4) as follow:

$$EU = 2qrXk + 2q(\alpha S_A + (1-\alpha)S_C)(1-k) + (2p+2q)(rX\delta_1 + (1-r)\delta_2). \quad (5)$$

Maximization of expected utility (5) must be carried out under the budget constraints, namely the repayment for impatient and patient investors in both regions do not exceed the liquidity plus half of cash-flow:

$$r\delta_1 \le \tfrac{1}{2}\alpha S_A(1-k) + \tfrac{1}{2}(1-\alpha)S_C(1-k) + k, \quad (6)$$
$$(1-r)\delta_2 \le \tfrac{1}{2}\alpha S_A(1-k) + \tfrac{1}{2}(1-\alpha)S_C(1-k). \quad (7)$$

To reduce possibility impatient investors leave the economy early in period t_1, the bank maximizes investors' expected utility by increasing as much as possible the short-term repayment, such that we may set $\delta_1 = \delta_2 = \delta$, and thus inequality budget constraints (6) and (7) can be replaced by their corresponding equality constraints:

$$r\delta = \tfrac{1}{2}(\alpha S_A + (1-\alpha)S_C)(1-k) + k, \quad (8)$$
$$(1-r)\delta = \tfrac{1}{2}(\alpha S_A + (1-\alpha)S_C)(1-k). \quad (9)$$

The optimal allocation problem is then defined as follows: find liquidity holding k, repayment δ, and ratio α such that maximize the investors' expected utility (5) in accordance with budget constraints (8) and (9).

3.2 Solution

Optimal allocation problem comprises two equality constraints and contains three unknown parameters k, δ, and α, and therefore can be solved by elimination. Since we have only equality constraints then any two parameters will always solve the optimization problem. Division (8) and (9) eliminates δ and then produces

$$\frac{r}{1-r} = 1 + \frac{2k}{(\alpha S_A + (1-\alpha)S_C)(1-k)}.$$

By solving the above equation for k, we obtain the optimal liquidity ratio k^*, i.e., proportion of funds should be invested in liquidity such that maximized the investors' expected utility:

$$k^*(\alpha) = \frac{(2r-1)(\alpha S_A + (1-\alpha)S_C)}{2(1-r) + (2r-1)(\alpha S_A + (1-\alpha)S_C)}. \tag{10}$$

From (8) and (9) we may write $r\delta = (1-r)\delta + k$, and hence we obtain the optimal repayment by the bank as follows:

$$\delta_1^*(\alpha) = \delta_2^*(\alpha) = \delta^*(\alpha) = \frac{k^*}{2r-1}, \tag{11}$$

where k^* is given by (10). Since $r \in (0,1)$, $\alpha \in [0,1]$, $S_A > 0$, and $S_C > 0$, then it can be easily verified that $k^* \in [0,1]$. Obviously, expressions for k^* in (10) and for δ^* in (11) still depend on an undetermined parameter α. Liquidity ratio k^* and repayment δ^* are always optimal since they satisfy the equality constraints (8) and (9).

Some interesting facts may be drawn from (10) and (11):

1. We may further write (10) as

$$k^*(\alpha) = \frac{k_1 + k_2\alpha}{k_3 + k_2\alpha},$$

where $k_1 = (2r-1)S_C$, $k_2 = (2r-1)(S_A - S_C)$, and $k_3 = 2(1-r) + (2r-1)S_C$. The effect of proportion α on the liquidity ratio and repayment can be identified by facts that

$$\frac{\partial k^*(\alpha)}{\partial \alpha} = \frac{k_2(k_3 - k_1)}{(k_3 + k_2\alpha)^2} > 0, \tag{12}$$

$$\frac{\partial \delta^*(\alpha)}{\partial \alpha} = \frac{1}{2r-1}\frac{\partial k^*(\alpha)}{\partial \alpha} > 0. \tag{13}$$

Based on (12) and (13) it can be stated that if we increase the portion of funds invested to productive project with higher return, i.e., increase α, then local bank should reserve more funds for liquidity and in the same time promises higher repayment to investors, i.e., increase k^* and δ^*.

2. It is assumed in the beginning that project S commits higher return if conducted in region A than in region C, i.e., $S_A > S_C$. What happen if region C may exhibit better improvement such that S_C approaching S_A? We have the followings:

$$\lim_{S_C \to S_A} k^*(\alpha) = \frac{(2r-1)S_A}{2(1-r) + (2r-1)S_A}, \tag{14}$$

$$\lim_{S_C \to S_A} \delta^*(\alpha) = \frac{S_A}{2(1-r) + (2r-1)S_A}. \tag{15}$$

The right-hand sides of (14) and (15) can be seen as the more competitive level of liquidity ratio and contract repayment. As S_C tends to S_A, the bank has more options for investing funds to projects with similar return.

With k^* and δ^* are given in (10) and (11), the optimal value of α can be specified by substituting them into expected utility (5) and then performing the first order condition of optimization. We firstly have

$$EU = \frac{2(\alpha S_A + (1-\alpha)S_C)((3q+p)(1-r) + rX(2qr+p))}{2(1-r) + (2r-1)(\alpha S_A + (1-\alpha)S_C)}. \tag{16}$$

Again we can easily verify that EU is a non-negative quantity. Note that expression (16) can be rewritten as a function of α as follows:

$$EU(\alpha) = \frac{m_1 + m_2\alpha}{n_1 + n_2\alpha},$$

where

$$\begin{gathered} m_1 = 2S_C\big((3q+p)(1-r) + rX(2qr+p)\big), \\ m_2 = 2\big((3q+p)(1-r) + rX(2qr+p)\big)(S_A - S_C), \\ n_1 = 2(1-r) + (2r-1)S_C, \\ n_2 = (2r-1)(S_A - S_C). \end{gathered}$$

By using simple calculus we may obtain the first partial derivative of EU with respect to parameter α as follows,

$$\frac{\partial EU(\alpha)}{\partial \alpha} = \frac{m_2 n_1 - m_1 n_2}{(n_1 + n_2\alpha)^2}.$$

Since $m_2 n_1 - m_1 n_2 = 4(1-r)(S_A - S_C)((3q+p)(1-r) + rX(2qr+p)) > 0$ by fact that $S_A > S_C$ as assumed in (2), then we conclude

$$\frac{\partial EU(\alpha)}{\partial \alpha} > 0. \tag{17}$$

The above condition reveals that the investors' expected utility EU is a monotonically increasing function with respect to α. It means that a larger α will contribute a bigger EU. As the bank wants to maximize EU, thus α should be selected as large as possible. Since α is defined as the proportion of funds invested in project S in region A and thus $0 \leq \alpha \leq 1$, then the optimal value of α is given by

$$\alpha^* = 1, \tag{18}$$

suggesting that funds allocated for project must all be used to finance project S in region A. Furthermore, with $\alpha^* = 1$ we may then strengthen the expression of optimal liquidity ratio (10) and that of optimal repayment δ^* (11) respectively as follow:

$$k^* = \frac{(2r-1)S_A}{2(1-r)+(2r-1)S_A}, \tag{19}$$

$$\delta^* = \frac{S_A}{2(1-r)+(2r-1)S_A}, \tag{20}$$

which show that optimal parameters mainly characterized by the return of project S in region A and the proportion of impatient investors in the population. We also have the maximum investors' expected utility by substituting (18) into (16):

$$EU^* = \frac{2S_A((3q+p)(1-r)+rX(2qr+p))}{2(1-r)+(2r-1)S_A}. \tag{21}$$

Note that expressions in (19) and (20) are exactly the same with those of (14) and (15). Liquidity ratio (19) and repayment (20) are derived by optimization process in the framework of allocation problem under budget constraints. Those of (14) and (15) are obtained by equating budget constraints and then changing the assumption from $S_C < S_A$ to $S_C = S_A$. Even if cases $S_C < S_A$ and $S_C = S_A$ suggest the same policies on liquidity and repayment, both offer distinct environments for integration. The former case recommends local bank to fully invest its funds in own region. The latter case, however, advises local bank to expand its business by financing project out of region. We refer case of two productive projects with identical return to a realistic environment for integration.

3.3 A realistic case

Our previous analysis shows that economic integration will not be realized when the two regions possess the same flagship project but offer different stream of returns. This fact, however, can be easily understood. If regions A and C have the same project S as their flagship, but it promises a higher return whenever run in region A than region C, then local bank in region A will only invest its funds in own region. There is no incentive for the bank to invest its funds in region C as it will reduce the repayment. In this situation an integration is impossible to be realized.

Let's consider a more ideal situation, where productive projects in both regions offer exactly the same level of return. More precisely, we consider an economy with two regions A and B and two projects S and R. In region A, project S is more productive than project R, and in region B, project R is more productive than project S. Both productive projects offer the same return and both inferior projects promise the same return. In other words, we assume

$$S_A = R_B, R_A = S_B, S_A > S_B, R_A < R_B. \tag{22}$$

This ideal case, which has been discussed in Fecht *et al.* (2012), proposes some sort of flexibility to invest. The bank may also invest funds in project R in region B, in addition to project S in region A. This setting, however, offers more room for integration between regions.

Instead of rigorously derived from beginning, we utilized some expressions in the previous case. We now denote by l the proportion of funds to be invested in liquidity. Out of $1-l$ of the available funds for financing project, some will be used for funding project S in region A with proportion β and the rest $1-\beta$ will be devoted for financing project R in region B. We also denote by d_1 and d_2 repayments in periods t_1 and t_2, respectively. By rearranging (5) we may have the following investors' expected utility to be maximized

$$\mathcal{EU} = 2qrXl + 2q(\beta S_A + (1-\beta)R_B)(1-l) + (2p+2q)(rXd_1 + (1-r)d_2). \tag{23}$$

We can simplify (23) by applying $S_A = R_B$ as assumed in (22) to get the following:

$$\mathcal{EU} = 2qrXl + 2qS_A(1-l) + (2p+2q)(rXd_1 + (1-r)d_2). \tag{24}$$

Note that we no longer have a parameter β in (23). This means funds allocation for project S in region A and project R in region B can be loosely decided as they have identical returns. In much the same way, we have also a set of budget constraints from (8)-(9):

$$rd = \tfrac{1}{2}S_A(1-l) + l, \tag{25}$$

$$(1-r)d = \tfrac{1}{2}S_A(1-l). \tag{26}$$

By dividing (25) and (26) we find the optimum value of liquidity ratio and repayment, respectively as follow:

$$l^* = \frac{(2r-1)S_A}{(2r-1)S_A + 2(1-r)}, \tag{27}$$

$$d_1^* = d_2^* = d^* = \frac{l^*}{2r-1} = \frac{S_A}{(2r-1)S_A + 2(1-r)}. \tag{28}$$

Note again that (27) and (28) can be reclaimed by applying condition $S_C = R_B$ and then $R_B = S_A$ into (10) and (11), respectively, with understanding that project S in region C is now project R in region B. Direct comparison of (10) and (27) provides

$$k^*(\alpha) - l^* = \frac{-2(2r-1)(1-r)(1-\alpha)(S_A - S_C)}{\big(S_A(2r-1) + 2(1-r)\big)\big(2(1-r) + (2r-1)(\alpha S_A + (1-\alpha)S_C)\big)}.$$

Since $\frac{1}{2} < r \leq 1$, $0 \leq \alpha \leq 1$, $S_A > S_C$, then $k^*(\alpha) - l^* \leq 0$, and thus we have the following relationships:

$$k^*(\alpha) \leq l^*, \tag{29}$$

$$\delta^*(\alpha) \leq d^*. \tag{30}$$

At optimal level, i.e., $\alpha = 1$, again we can show that the liquidity ratio and repayment in both cases are the same, i.e., $k^* = l^*$ and $\delta^* = d^*$.

If we substitute (27) and (28) into (24), the we have the maximum investors' expected utility for ideal case

$$\mathcal{EU}^* = \frac{2S_A((3q+p)(1-r) + rX(2qr+p))}{2(1-r) + (2r-1)S_A}, \tag{31}$$

and again (31) is exactly the same with that of previous case given in (21). These facts reveal that even though we consider different environments, one without incentive for integration and one promotes integration, both cases recommend exactly the same way of funds allocation and contract repayments.

Furthermore, for both cases we also have the followings:

$$\frac{dk^*}{dS_A} = \frac{dl^*}{dS_A} = \frac{2q-1}{(2(1-q)+(2q-1)S_A)^2} > 0, \tag{32}$$

$$\frac{d\delta^*}{dS_A} = \frac{dd^*}{dS_A} = \frac{1}{(2(1-q)+(2q-1)S_A)^2} > 0, \tag{33}$$

which assert that the liquidity ratio and repayment are increasing with respect to project return. If the return of project increase, then the bank can pay more to investors but at the same time it should invest more funds to ~~storage technology~~ liquidity?

4. Conclusion

We have examined an allocation problem of investment contract in two-region economy. Two different conditions of contract were considered to analyze the bank investment decision making in short-term liquidity and long-term projects. In the first case, two regions (Indonesia and Malysia) have similar productive projects but different stream of returns. While in the second case, two regions have different productive projects but identical stream of returns. Under assumption that the bank invests only in productive projects, it has been revealed the following facts:

1. In the optimal level, both cases provide exactly the same amounts of liquidity ratio, contract repayment and expected utility. The first two parameters are entirely determined by the return of productive project and the ratio of impatient and patient investors. The expression of maximal expected utility depends also on the probability of shock and the marginal utility of private investment.
2. In the first case, it is suggested that funds devoted to financing long-term project should all be invested in productive project in the same region of bank. While in the second case, the bank has flexibility to invest the funds not only in productive project in the same region but also in another region as cross-border activities are allowed in this case.
3. In order to promote the similar room of cross-border activities, it is recommended for region with lower project's return to increase the return such that it has a comparable return. Cross-border activities can be seen as incentives for integration between the two economies. The increase in return will be followed by an increase in repayment by bank, although in this situation bank must raise the portion of funds invested in liquidity.
4. The result that the equality in return may open the possibility of cross-border risk sharing shows that there is a potential advantage from government intervention into interbank markets implementation.

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